

Time-dependent solutions of the Fokker–Planck equation of maximally reduced air–sea coupling climate model

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Abstract

The time-dependent solutions of a reduced air–sea coupling stochastic-dynamic model are obtained using the Fokker–Planck equation and the quantum mechanical method. The analysis of the time-dependent solutions suggests that when the climate system is in the ground state, the system behaves in a Brownian motion, thus theoretically revealing the foothold of Hasselmann’s stochastic climate model; when the system is in the first excitation state, it evolves in a form of time-decaying, or a 2.3-year periodic oscillation under certain condition. At last, the results are used to discuss the impact of the doubling of carbon dioxide concentration on climate system.

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1. Introduction

Air–sea interaction is an important physical process in the climate system. Oceans cover two thirds of earth surface, and have a tremendous thermal inertia, therefore oceans exercise an extremely important influence on atmospheric motion, and the air–sea interaction becomes a core item in climate change studies. In turn, the atmosphere affects the motion of seawater through wind-driven current and heat transfer. At the same time, the ocean is a most important reservoir for carbon dioxide, and it absorbs about 1×10^{12} – 3×10^{12} kg of carbon from the atmosphere per year, which accounts for 40–50% of the total carbon dioxide emission from fossil fuel burning. Therefore, scientists not only analyzed observational facts to reveal the coupling between the atmosphere and ocean, but also developed air–sea coupling models with different complex extents to investigate the physical process [1–4]. Li and Huang [5] derived a two-variable linear coupling model from the parameterization of diabatic process, and discussed the spectral characteristics of the stochastic-dynamical model. Feng et al. [6] discussed the approximate solution of the model, and introduced the effect of carbon dioxide into the model. A non-deterministic cell dynamical system model for atmospheric flows was devel-

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oped for the self-organized criticality as being intrinsic to quantum-like mechanics governing flow dynamics [7–9]. In this paper, we apply the eigenfunction solution principle in quantum mechanics, yielding the time-dependent solutions of the model. Then the analytical solution is used to investigate the greenhouse effect of carbon dioxide doubling and the change in the behavior of the model, yielding some meaningful results.

2. The steady-state solution

The air–sea coupling stochastic-dynamic model equations [5,6] are

$$\frac{dT}{dt} = -\alpha_1 T + \beta_1 T_s + \omega_1(t), \tag{1}$$

$$\frac{dT_s}{dt} = -\alpha_2 T_s + \beta_2 T + \omega_2(t), \tag{2}$$

where $T = \tilde{T} - \bar{T}, T_s = \tilde{T}_s - \bar{T}_s$ are the deviations of air temperature \tilde{T} , and sea surface temperature (SST) \tilde{T}_s , from equilibrium state \bar{T} and \bar{T}_s , respectively. α_1, α_2 and β_1, β_2 are self-feedback coefficients and mutual-feedback coefficients, respectively, and they are associated with the parameters such as solar constant, effective emittance (ε), specific heat at constant pressure, densities of air and water, the exchange coefficient of sensible heat flux, and so on [5,6]. The effect of carbon dioxide is introduced into the model through its effective emittance $\varepsilon_c = a \ln[\text{CO}_2] + b$, that is to say, $\varepsilon = \varepsilon_c + \varepsilon^*$, where ε^* is the integrated effective emittance of all gases except CO_2 , a and b are empirical coefficients ($a = 0.0235, b = 0.0537$); $\omega_1(t)$ and $\omega_2(t)$ are stochastic noises, and both assumed to be the white noises of zero mean value for convenience, i.e.

$$\begin{aligned} \langle \omega_1(t) \rangle &= \langle \omega_2(t) \rangle = 0, \\ \langle \omega_1(t) \omega_1(t') \rangle &= 2q \delta(t - t'), \\ \langle \omega_2(t) \omega_2(t') \rangle &= 2q \delta(t - t'), \end{aligned} \tag{3}$$

where q is the intensity of noise, and δ the Dirac function. Differentiating Eq. (2) with respect to t , and eliminating T and dT/dt by using Eq. (1) yield

$$\frac{d^2 T_s}{dt^2} + \gamma \frac{dT_s}{dt} + \omega_0^2 T_s = \omega(t), \tag{4}$$

where $\gamma = \alpha_1 + \alpha_2, \omega_0^2 = \alpha_1 \alpha_2 - \beta_1 \beta_2 \geq 0$, and $\omega(t) = \alpha_1 \omega_2(t) + \beta_2 \omega_1(t)$.

2.1. Fokker–Planck equation

In order to solve Eq. (4) accurately, it is transformed into a Langevin equation group

$$\begin{aligned} \dot{T}_s &= v, \\ \dot{v} &= -\gamma v - f'(T_s) + \omega(t), \end{aligned} \tag{5}$$

where $f'(T_s) = \omega_0^2 T_s$, then Eq. (5) may be written in the form of Fokker–Planck equation [10–15]

$$\begin{aligned} \frac{\partial W}{\partial t} &= L_k W, \\ L_k &= L_k(T_s, v) = -\frac{\partial}{\partial T_s} v + \frac{\partial}{\partial v} [\gamma v + f'(T_s)] + q \frac{\partial^2}{\partial T_s^2}, \end{aligned} \tag{6}$$

where W is the probability density distribution function of T, v, t , and L the operator of the Fokker–Planck equation. $f(T_s)$ is a potential function, and $f'(T_s)$ is its derivative. Eq. (6) can be further written in the form of the continuity equation of probability flow:

$$\frac{\partial W}{\partial t} + \frac{\partial S_{T_s}}{\partial T_s} + \frac{\partial S_v}{\partial v} = 0, \tag{7}$$

where

$$S_{T_s} = T_s W, \quad S_v = -[\gamma v + f'(T_s)] W - q \frac{\partial W}{\partial T_s}. \tag{8}$$

2.2. Steady-state solution

The steady-state solution can be obtained when and only when probability flows S_{T_s}, S_v equal to constants everywhere for T_s, v , i.e. not changing with time. As known from Eq. (8), if $\partial S_v / \partial v = 0$ is what we wanted, expression (9) is valid

$$q \frac{\partial W}{\partial T_s} = -[\gamma v + f'(T_s)]W. \tag{9}$$

Then, distribution function $W(T_s, v)$ is obtained via integrating Eq. (7) with respect to time

$$W(T_s, v) = A \cdot \exp \left[-\frac{\gamma}{2q} (v^2 + \omega_0^2 T_s^2) \right], \tag{10}$$

where A is determined by normalization condition

$$\int W(T_s, v) dT_s dv = 1. \tag{11}$$

Thus, the results, same as Ref. [6], are derived through different paths.

3. Time-dependent solution

According to the definition of the first-order moment in probability theory, the mean value of T_s can be calculated from Eq. (10)

$$\langle T_s \rangle = \int T_s W(T_s, v) dT_s dv. \tag{12}$$

However, the temporal evolution of $\langle T_s \rangle$ is still unknown, therefore it is necessary to get the unsteady state solution (time-dependent solution) for Eq. (6). The Fokker–Planck equation operator in Eq. (6) can be written as the sum of a reversible and an irreversible operator

$$L_k = L_{rev} + L_{ir}, \tag{13}$$

$$L_{rev} = -v \frac{\partial}{\partial T_s} + f'(T_s) \frac{\partial}{\partial v}, \tag{14}$$

$$L_{ir} = \gamma \frac{\partial}{\partial v} \left(v + \frac{q}{\gamma} \frac{\partial}{\partial v} \right). \tag{15}$$

Generally, L_k, L_{rev} and L_{ir} are not Hermitian operators, therefore it is difficult to get its eigenvalue and corresponding eigenfunction. However, we can use suitable transforms to quantize them, i.e. after transformation, their forms are Hermitian operators. As known from the steady-state solution, the distribution of L_{ir} is positively proportional to $\exp\left(-\frac{\gamma v^2}{2q}\right)$. After L_{ir} operator is left multiplied by $\exp\left(\frac{\gamma v^2}{4q}\right)$ as well as right multiplied by $\exp\left(-\frac{\gamma v^2}{4q}\right)$, it can be proved that \bar{L}_{ir} be a Hermitian operator

$$\bar{L}_{ir} = \exp\left(\frac{\gamma v^2}{4q}\right) L_{ir} \exp\left(-\frac{\gamma v^2}{4q}\right) = \gamma \left[\frac{q}{\gamma} \frac{\partial^2}{\partial v^2} - \frac{1}{4} \left(\frac{\gamma v^2}{q} \right) + \frac{1}{2} \right] = \bar{L}_{ir}^+. \tag{16}$$

Further, similar to Ref. [16] and [17], we may introduce the eigenfunction and eigenvalue for \bar{L}_{ir} operator. Therefore, define Boson-operators

$$b = \sqrt{\frac{q}{\gamma}} \frac{\partial}{\partial v} + \frac{1}{2} \frac{v}{\sqrt{\frac{q}{\gamma}}}, \tag{17}$$

$$b^+ = -\sqrt{\frac{q}{\gamma}} \frac{\partial}{\partial v} + \frac{1}{2} \frac{v}{\sqrt{\frac{q}{\gamma}}}$$

and their commutation relation is

$$[b, b^+] = 1, \tag{18}$$

then

$$\bar{L}_{ir} = -\gamma b^+ b. \tag{19}$$

Expressions (18) and (19) are the well-known Hamilton’s form of simple harmonic oscillators. Similarly, a suitable transform for \bar{L}_{rev} yields

$$\bar{L}_{rev} = -v \frac{\partial}{\partial T_s} + f'(T_s) \frac{\partial}{\partial v} - \frac{1}{2} f''(T_s) \frac{v\gamma}{q} = L_{rev} - \frac{1}{2} f'' \frac{\gamma}{q} = -bD - b^+D, \tag{20}$$

where

$$D = \sqrt{\frac{q}{\gamma}} \frac{\partial}{\partial T_s}, \quad \hat{D} = \sqrt{\frac{q}{\gamma}} \frac{\partial}{\partial T_s} + f'(T_s) \sqrt{\frac{q}{\gamma}}, \quad [D, \hat{D}] = f'' = \omega_0^2. \tag{21}$$

Thus, the Fokker–Planck equation operator may be written as

$$\bar{L}_k = \bar{L}_{rev} + \bar{L}_{ir}, \tag{22}$$

$$\bar{L}_k = -\gamma b^+ b - bD - b^+D. \tag{23}$$

Since time t is not explicitly contained in $f(T_s)$, distribution function $W(T_s, v, t)$ may be written as

$$W(T_s, v, t) = W_\lambda(T_s, v) \cdot f_\lambda(t). \tag{24}$$

With similar treatment, the eigenvalue and corresponding eigenfunction of Fokker–Planck equation operator are

$$\varphi(T_s, v, t) = \varphi(T_s, v) e^{-\lambda t}, \tag{25}$$

$$L_k \varphi(T_s, v) = -\lambda \varphi(T_s, v) \tag{26}$$

and their transformed forms are

$$\bar{L}_k \bar{\varphi}(T_s, v) = -\lambda \bar{\varphi}(T_s, v), \tag{27}$$

$$\bar{\varphi}(T_s, v) = \exp \left[\frac{\gamma}{4q} (v^2 + f(T_s)) \right] \varphi(T_s, v). \tag{28}$$

Since the commutation of D and \hat{D} is ω_0^2 , therefore define the Boson-operator as

$$\alpha = \frac{\hat{D}}{\omega_0} = \frac{\sqrt{q}}{\omega_0} \frac{\partial}{\partial T_s} + \frac{\omega_0}{2\sqrt{q}} T_s, \tag{29}$$

$$\alpha^+ = -\frac{D}{\omega_0} = -\frac{\sqrt{q}}{\omega_0} \frac{\partial}{\partial T_s} + \frac{\omega_0}{2\sqrt{q}} T_s.$$

α and α^+ satisfy the commutation relation $[\alpha, \alpha^+] = 1$, then

$$\bar{L}_k = -\gamma b^+ b - \omega_0 (a b^+ - a^+ b). \tag{30}$$

As known from its steady-state solution, $\lambda = 0, \bar{\varphi}_{0,0}$ can be obtained from the following expressions:

$$\begin{aligned} \alpha \bar{\varphi}_{0,0} &= b \bar{\varphi}_{0,0} = 0, \\ \bar{\varphi}_{0,0}(T_s, v) &= \sqrt{\frac{\omega_0 \gamma}{2\pi q}} \exp \left[-\frac{\gamma}{4} \left(\frac{v^2}{q} + \frac{\omega_0^2 T_s^2}{q} \right) \right] \end{aligned} \tag{31}$$

and its corresponding probability distribution is

$$W(T_s, v) = |\varphi_{0,0}(T_s, v)|^2 = \frac{\omega_0 \gamma}{2\pi q} \exp \left[-\frac{\gamma}{2q} (v^2 + \omega_0^2 T_s^2) \right], \tag{32}$$

which is consistent with that of steady-state solution. The value of $\langle T_s \rangle$ is calculated from Eq. (12) using the values of parameters $\alpha_1, \alpha_2, \beta_1, \beta_2, q$ in Ref. [5],

$$\langle T_s \rangle = \int T_s W(T_s, v) dT_s dv = 2.71 \text{ }^\circ\text{C} \tag{33}$$

and this result is in accord with the SST anomaly fluctuations within the range of 0.5–3.5 °C.

In order to rewrite Eq. (30) as a simpler form, a set of new Boson-operators are introduced

$$\begin{aligned} C_1^+ &= \delta^{-\frac{1}{2}}\left(\sqrt{\lambda_1}b^+ - \sqrt{\lambda_2}\alpha^+\right), & C_1^- &= \delta^{-\frac{1}{2}}\left(\sqrt{\lambda_1}b + \sqrt{\lambda_2}\alpha\right), \\ C_2^+ &= \delta^{-\frac{1}{2}}\left(-\sqrt{\lambda_2}b^+ + \sqrt{\lambda_1}\alpha^+\right), & C_2^- &= \delta^{-\frac{1}{2}}\left(\sqrt{\lambda_2}b + \sqrt{\lambda_1}\alpha\right), \end{aligned} \tag{34}$$

where λ_1 and λ_2 are the eigenvalues of the equation when no noise exists [10],

$$\lambda_1 = (\gamma + \delta)/2, \quad \lambda_2 = (\gamma - \delta)/2, \tag{35}$$

$$\delta = \sqrt{\gamma^2 - 4\omega_0^2} = \lambda_1 - \lambda_2. \tag{36}$$

Thus, \bar{L}_k becomes the sum of two simple harmonic vibrator Hamilton's operators

$$\bar{L}_k = -\lambda_1 C_1^+ C_1^- - \lambda_2 C_2^+ C_2^-. \tag{37}$$

The commutation relations of $C_1^+, C_1^-, C_2^+, C_2^-$ are as follows:

$$\begin{aligned} [C_1^+, C_1^-] &= [C_2^+, C_2^-] = 1, \\ [C_1^-, C_2^+] &= [C_2^-, C_1^+] = [C_1^-, C_2^-] = [C_1^+, C_2^+] = 0, \\ [(-\bar{L}_k), C_i^\pm] &= \pm\lambda_i C_i^\pm \quad (i = 1, 2). \end{aligned} \tag{38}$$

λ_{n_1, n_2} and corresponding eigenfunctions can be obtained from quantum mechanics

$$\bar{\varphi}_{n_1, n_2}(T_s, v) = (n_1!n_2!)^{-\frac{1}{2}}(C_1^+)^{n_1}(C_2^+)^{n_2}\bar{\varphi}_{0,0}(T_s, v), \tag{39}$$

$$\lambda_{n_1, n_2} = \lambda_1 n_1 + \lambda_2 n_2 = \frac{1}{2}\gamma(n_1 + n_2) + \frac{1}{2}\delta(n_1 - n_2), \tag{40}$$

$$\bar{\varphi}_{n_1, n_2}^*(T_s, v) = (n_1!)^{-\frac{1}{2}}(n_2!)^{-\frac{1}{2}}(C_1^-)^{n_1}(C_2^-)^{n_2}\bar{\varphi}_{0,0}(T_s, v), \tag{41}$$

$$W_\lambda(T_s, v) = |\bar{\varphi}_{n_1, n_2}^* \cdot \bar{\varphi}_{n_1, n_2}|, \tag{42}$$

where n_1 and n_2 are positive integers. The probability distribution function W_λ expressed in terms of eigenfunction $\bar{\varphi}$ can be obtained from Eqs. (39)–(42), and the mean temperature anomaly $\langle T_s \rangle$ can be calculated from Eq. (12). Eqs. (37)–(42) are obtained without the condition of $\partial W/\partial t = 0$, therefore W and T_s are both time-dependent solutions.

4. Results and discussions

(1) If $N = n_1 + n_2 = 0$, i.e. $n_1 = 0, n_2 = 0$. This is right a steady-state solution, i.e. the ground state, which represents the Brownian movement. Therefore, the point of view of Hasselmann's stochastic climatic model, viewing climate change as a Brownian movement of light and heavy particles collision [18], is theoretically proved in this paper. Viewed from the aspect of quantum mechanics, the possible behaviors of oceans are mostly in the ground state, and the SST anomaly fluctuates around its mean value 2.71 °C within the range of 0.5–3.5 °C. Disturbed by factors such as sunspots, volcanic eruption, polar ice changes, etc., the air–sea system may transit from the ground state to the first excited state or secondary excited state or even higher order excited state. Since the distribution function is positively proportional to $\exp(-\lambda t)$, the higher the order of the excited state is, the shorter the state lasts. Considering that the higher order excited state has less opportunity to occur and decays rapidly, we discuss only the first excited state here.

(2) If $N = n_1 + n_2 = 1$, i.e. $n_1 = 0, n_2 = 1$ or $n_1 = 1, n_2 = 0$, the eigenvalues of the system are

$$\lambda_{0,1} = \frac{1}{2}(\gamma - \delta) = \lambda_1, \quad \lambda_{1,0} = \frac{1}{2}(\gamma + \delta) = \lambda_2. \tag{43}$$

Therefore, λ_1, λ_2 are only the particular cases of λ_{n_1, n_2} .

When and only when $\gamma \gg 2\omega_0$, namely, only when the self-feedback coefficient is permanently greater than the mutual-feedback one, λ_{n_1, n_2} can be reduced to

$$\lambda_{n_1, n_2} = \gamma_{n_1} + (\omega_0^2/\gamma)(n_2 - n_1) + O(\gamma^{-3}). \tag{44}$$

As known from Eq. (36),

$$\delta = (\gamma^2 - 4\omega_0^2)^{\frac{1}{2}} \approx \gamma, \tag{45}$$

therefore,

$$\lambda_{0,1} = \lambda_1 \approx 0, \quad \lambda_{1,0} = \lambda_2 \approx \gamma. \tag{46}$$

The eigenfunctions of the first excited state are

$$\begin{aligned} \bar{\varphi}_{1,0}(T_s, v, t) &= (C_1^+) \bar{\varphi}_{0,0}(T_s, v) e^{-\lambda_2 t} \\ &= -\delta^{-\frac{1}{2}} \sqrt{\lambda_2} \alpha^+ \bar{\varphi}_{0,0}(T_s, v) e^{-\lambda_2 t}, \end{aligned} \tag{47}$$

$$\bar{\varphi}_{1,0}^*(T_s, v, t) = \delta^{-\frac{1}{2}} \sqrt{\lambda_2} \alpha^+ \bar{\varphi}_{0,0}(T_s, v) e^{-\lambda_2 t}. \tag{48}$$

Then, the possible probability distribution $W(T_s, v, t)$ of the first excited state is

$$W(T_s, v, t) = |\varphi_{1,0}(T_s, v, t)| = \varphi_{1,0}^*(T_s, v, t) \varphi_{1,0}(T_s, v, t) = A \cdot \exp\left[-\frac{\gamma}{2q}(v^2 + \omega_0^2 T_s^2)\right] T_s^2 e^{-2\lambda_2 t} \tag{49}$$

and

$$W(T_s, t) = \int W(T_s, v, t) dv = B \cdot \exp\left(\frac{\gamma}{2q} \cdot \omega_0^2 T_s^2\right) T_s^2 e^{-2\lambda_2 t}, \tag{50}$$

where A and B are normalized coefficients. Eqs. (49) and (50) essentially differ from the steady ground state in the time-decaying factor of $\exp(-\lambda t)$. Based on Eqs. (12) and (50), the mathematic expectation of SST anomaly T_s is

$$\langle T_s \rangle = \frac{\int_0^\infty W(T_s, t) T_s dt}{\int_0^\infty W(T_s, t) dt} = \frac{2}{\sqrt{\frac{\gamma}{2q} \omega_0^2 \pi}} e^{-2\lambda_2 t}. \tag{51}$$

With carbon dioxide concentration be 330×10^{-6} , calculated $\alpha_1, \alpha_2, \beta_1, \beta_2, q$ are 0.0001, 0.5127, 0.2186, -0.03921 (RAD month⁻¹), and 1.5 (RAD year⁻¹), respectively; when carbon dioxide concentration doubled, calculated $\alpha_1, \alpha_2, \beta_1, \beta_2, q$ respectively become 0.0982, 0.5126, 0.2186, 0.3919 (RAD month⁻¹), and 1.5 (RAD month⁻¹), then

$$\langle T_s \rangle_{330} = 0.068 e^{-0.96t}, \tag{52}$$

$$\langle T_s \rangle_{660} = 0.192 e^{-0.71t}. \tag{53}$$

When carbon dioxide concentration doubled, and under the first excited state, the increment of the mean value for SST anomaly calculated from Eqs. (52) and (53) is about 0.12 °C; while under the ground state, the corresponding increment

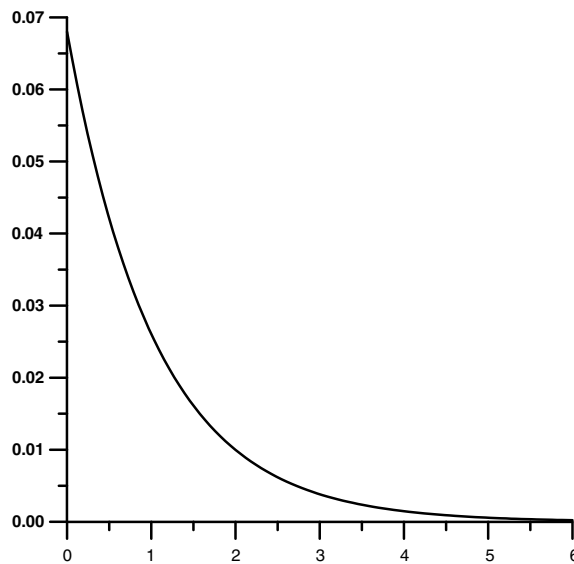


Fig. 1. Decaying of SST deviation (T_s) with time when the concentration of CO_2 is 330×10^{-6} (the abscissa: month).

is 1.81 °C. However, the interesting thing is 0.71 and 0.96 in the exponential term. That is to say, when the carbon dioxide concentration is doubled, the decay becomes slow, which is a meaningful result. As shown in Figs. 1 and 2, the doubling of CO₂ concentration not only makes oceans warm, but also extends its decay period, indicating that oceans have a bigger thermal inertia. This result accords with physical consideration.

In general, delay time is used to compare the extent of decay, here $\tau_{330} = 1/0.96 = 1.04$ (month), $\tau_{660} = 1/0.71 = 1.41$ (month), and $\tau_{660} > \tau_{330}$. When the CO₂ concentration is 330×10^{-6} , it takes 1.04 months for $\langle T_s \rangle$ to decay to 37% of its original value; but when the CO₂ concentration is 660×10^{-6} , it takes 1.41 months. As known from Eqs. (35) and (40), when $\gamma < 2\omega_0$, λ_{n_1, n_2} is a complex number

$$\begin{aligned} \delta &= i\sqrt{4\omega_0^2 - \gamma^2} = 2i\omega, \\ \lambda_{n_1, n_2} &= \frac{1}{2}\gamma(n_1 + n_2) + \frac{1}{2}2i\omega(n_1 - n_2), \end{aligned} \tag{54}$$

and the probability distribution function puts up periodic oscillations with the period being $\frac{2\pi}{2\omega} = \frac{\pi}{\omega}$. The eigenvalues of the first excited state are

$$\lambda_{1,0} = \frac{1}{2}(\gamma + i2\omega), \quad \lambda_{0,1} = \frac{1}{2}(\gamma - i2\omega) \tag{55}$$

and their oscillation forms are consistent with each other, consequently only the form of $\lambda_{1,0} = \frac{1}{2}(\gamma + i2\omega)$ is discussed. The eigenfunctions are

$$\begin{aligned} \bar{\varphi}_{1,0}(T_s, v, t) &= \delta^{-\frac{1}{2}}(\sqrt{\lambda_1}b^+ - \sqrt{\lambda_2}\alpha^+)\bar{\varphi}_{0,0}(T_s, v) \\ &\approx (Av + BT_s)e^{-\frac{\gamma}{4q}(v^2 + \omega_0^2 T_s^2)} e^{-\frac{1}{2}\gamma t} e^{-i\omega t}, \end{aligned} \tag{56}$$

$$\bar{\varphi}_{1,0}^*(T_s, v, t) \approx (Av + BT_s)e^{-\frac{\gamma}{4q}(v^2 + \omega_0^2 T_s^2)} e^{-\lambda_{1,0} t}, \tag{57}$$

where A, B are coefficients, determined by normalization condition. The following expressions can be obtained from Eqs. (49) and (51)

$$\langle T_s \rangle_{330} = 0.97e^{-0.5127t} e^{-i0.2243t}, \tag{58}$$

$$\langle T_s \rangle_{660} = 1.82e^{-0.5126t} e^{-i0.2153t}. \tag{59}$$

It can be seen from Eqs. (58) and (59) that when the CO₂ concentration is doubled, the increment of SST is 0.85 °C. The temporal variation of the real part of $\langle T_s \rangle_{330}$ is close to that of $\langle T_s \rangle_{660}$; while the period $T_{330} = \frac{2\pi}{0.2243} = 2.3$ years and

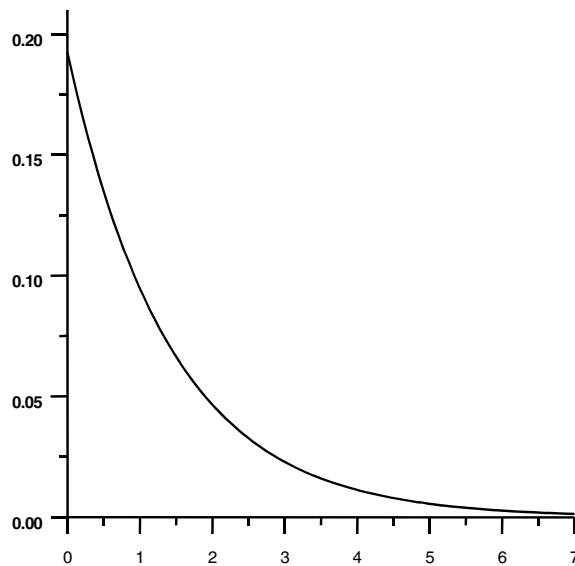


Fig. 2. Same as Fig. 1 but for the concentration of CO₂ being 660×10^{-6} .

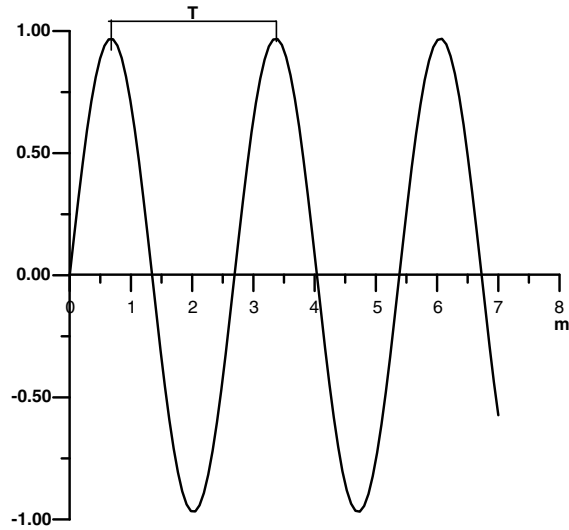


Fig. 3. The periodical change of T_s with time when the concentration of CO_2 is 330×10^{-6} (T is the period; the abscissa: month).

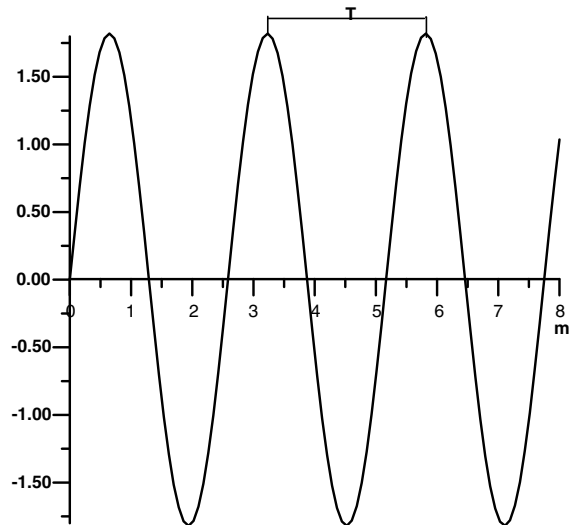


Fig. 4. Same as Fig. 3 but for the concentration of CO_2 being 660×10^{-6} .

$T_{660} = \frac{2\pi}{0.2153} = 2.4$ years, shown in Figs. 3 and 4. This result can be used to explain the quasi-biennial periodic oscillations in the atmosphere and oceans.

It can be clearly seen from Figs. 3 and 4 that when the CO_2 concentration is doubled, the oscillation period is prolonged, although not so much. This indicates that under the influence of the increment of SST induced by CO_2 doubling, the periodic response time of the air–sea coupling system to external disturbances will be prolonged, which remains to be confirmed by other simulation results.

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